The Calculation of TEM-, TE- and TM-Waves in Shielded Strip Transmission Lines

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For the systematic development of new TEM-microwave components the accurate calculation of the TEM-wave characteristic impedance is necessary. Besides the characteristic impedance the properties of TE- and TM-modes in new configurations are important. By TE- or TM-modes in a device designed for TEM-waves the function of this device is gravely disturbed. As the probability of TE- or TM-modes growths with frequency increasing this question is important especially at microwave frequencies.

Many of the modern transmission line structures are accessible only to numerical calculation methods. Therefore I generated computer programs for the calculation of TEM-, TE- and TM-waves in transmission lines with general cross sections. In the following, after a short description of the numerical methods, results for the special case of shielded strip transmission lines with rectangular inner conductor and rectangular outer conductor will be presented. The TEM-wave characteristic impedance can be calculated if the Laplace equation has been solved for the proper boundary conditions. For the solution I used a finite difference method. The application of this method was demonstrated in /1, 2, 3/. However, some accuracy problems remained unsolved. Therefore I made some investigations concerning the accuracy of the method. For the calculation of TE- and TM-modes an advanced version of a method described in /4/ was employed. This method applies the static field lines and the static equipotential lines as a curvilinear coordinate system. These lines can be easily obtained together with the TEM-wave characteristic impedance. All numerical calculations were performed at the TR4-computer of the Bavarian Academy of Sciences, Munich.

The Finite Difference Method for the Solution of the Laplace Equation and the Error of Discretization. The cross section of the transmission line is superposed by a square mesh. The finite difference method is based upon the assumption that the potential in every mesh point is approximately the arithmetic mean of the potentials in the four neighbouring mesh points. This assumption is appropriate whenever the derivatives of the potential function with orders higher than two are negligibly small. The unknown static potentials in the inner mesh points can be calculated from an inhomogeneous linear equation system by an iteration process called relaxation /1, 2, 3/. The method involves an error of discretization and a residual error of relaxation. The latter is a consequence of the iteration process and has inferior importance as it can be made sufficiently small by a reasonable number of iteration runs. The error of discretization decreases with the mesh number N increasing. For N there are set limits by the available computer storage. Therefore it is important to know the dependence of this error upon N to select the appropriate N for a given problem and to gain higher accuracy by extrapolation to N infinite.

For shielded strip transmission lines I studied the error of discretization by help of numerical investigations. In these transmission lines at the 90°-edges of the inner conductor all derivatives of the potential function are infinite.

Therefore these edges are the main sources of the discretization error. Up to a constant factor the dependence of the discretization error on N is about the same for all shielded strip transmission lines due to these edges. For my investigations I chose examples with relatively large inner conductors for which I could calculate the characteristic impedance by conformal transformation /3/ with an accuracy far exceeding the accuracy obtainable by the finite difference method. After the quasi-exact calculation by conformal transformation I treated the same examples by the finite difference method with various mesh numbers N. The comparison of the results of both methods immediately gave the error of discretization, depending on N.

The finite difference method primarily yields the static potentials in the mesh points. From these the characteristic impedance can be calculated via the dielectric displacement on the inner conductor (case I), /1, 2/, or on the outer conductor (case II), /1, 2/, or via the total electrical field energy (case III), /3/. In fig. 1 for a representative example the relative error of the characteristic impedance (cases I, II, III) is drawn depending on N. It can be seen that the energy method (case III) is the most accurate. A more detailed study of fig. 1 allows the generally valid conclusion that with 90°-edges at the inner conductor the error of discretization is approximately a linear function of N^{-n} with $n \leq 1.5$. In the literature sometimes the too optimistic value $n = 2$ is suggested. In /3/ it is shown that Lagrange extrapolation to infinite N with $n = 1.25$ effects an improvement of the accuracy by about one order.

In fig. 2 the characteristic impedance of shielded strip transmission lines is drawn depending on the width r of the inner conductor.

The Calculation of TE- and TM-Modes. For the cross section a curvilinear coordinate system is established, see fig. 3. This results from the solution of the Laplace equation which is necessary for the calculation of the TEM-wave characteristic impedance. The coordinate system gives the conformal transformation of the original cross section upon the cross section of a parallel plate line with equal characteristic impedance. In the curvilinear coordinate system the metallic boundaries of the cross section coincide with lines of constant coordinate values (outer conductor $y = 0$, inner conductor $y = b$). This is convenient for the formulation of the boundary conditions. In the curvilinear coordinate system the field intensities of the TE- and TM-modes are performed by trigonometric series expansions with coefficients to be calculated. If the series expansions are inserted into Maxwell's equations there result linear equations for the coefficients. These equations represent a matrix eigenvalue problem. By means of the equations the critical wavelengths and the unknown coefficients can be calculated. From these all wanted informations about the waveguide modes can be obtained by simple numerical procedures. The method is applicable to any transmission line for the cross section of which the static field an equipotential lines are known. A more detailed explanation of the theory is to be found in /3/. It will be published in the near future.

In the following I will present some examples of my calculations . More examples are collected in /3/. Fig. 4 shows the transverse electric resp. magnetic field patterns for various TE- resp. TM-modes. In this case the dimensions of the transmission line are $p/q = 1.5$; $s/q = 0.2$; $r/p = 0.5$. The names of the modes indicate to which modes in the rectangular waveguide without inner conductor they are related. In figs. 5a,b the critical wavelengths for the most important modes are drawn depending on the width r of the inner conductor.

In order to test the accuracy of the calculations some critical wavelengths were controlled by resonator measurements. The resonators had the cross sectional dimensions

1) $p/q = 1.5$; $r/p = 0.75$; $s/q = 0.25$
2) $p/q = 1.5$; $r/p = 0.50$; $s/q = 0.25$.

The mechanical tolerances of the resonators were better than 0.1%. In the following table we see the good coincidence of the measurements and of the calculations.

Mode	normalized critical wavelength λ_c/p			
	Resonator 1		Resonator 2	
	measured	calculated	measured	calculated
TE_{10}	2.165	2.1654	2.244	2.2425
TE_{30}	0.729	0.7339	0.680	0.6819
TE_{01}	2.791	2.7961	2.015	2.0170
TE_{11}	1.382	1.3744	1.188	1.1785
TM_{11}	0.485	0.4852	0.535	0.5342

Literature

/1/ H.E.Green, The Numerical Solution of Some Important Transmission Line Problems, IEEE Trans., vol. MTT-13, no.3, Sept. 1965

/2/ M.V. Scheider, Computation of Impedance and Attenuation of TEM-Lines by Finite Difference Methods, IEEE Trans., vol. MTT-13, no. 3, Nov. 1965

/3/ W.Baier, F.R. Holtzhaussen, A. Mirus, O.Pfaffenberger, Unpublished Notes, Institute for High Frequency Techniques, Technical High School, Munich, 1965-1967

/4/ W.Baier and H.H.Meinke, The Characteristics of Waveguides with General Cross Section, NTZ-Communications Journal 7 (1968), no. 1, being printed

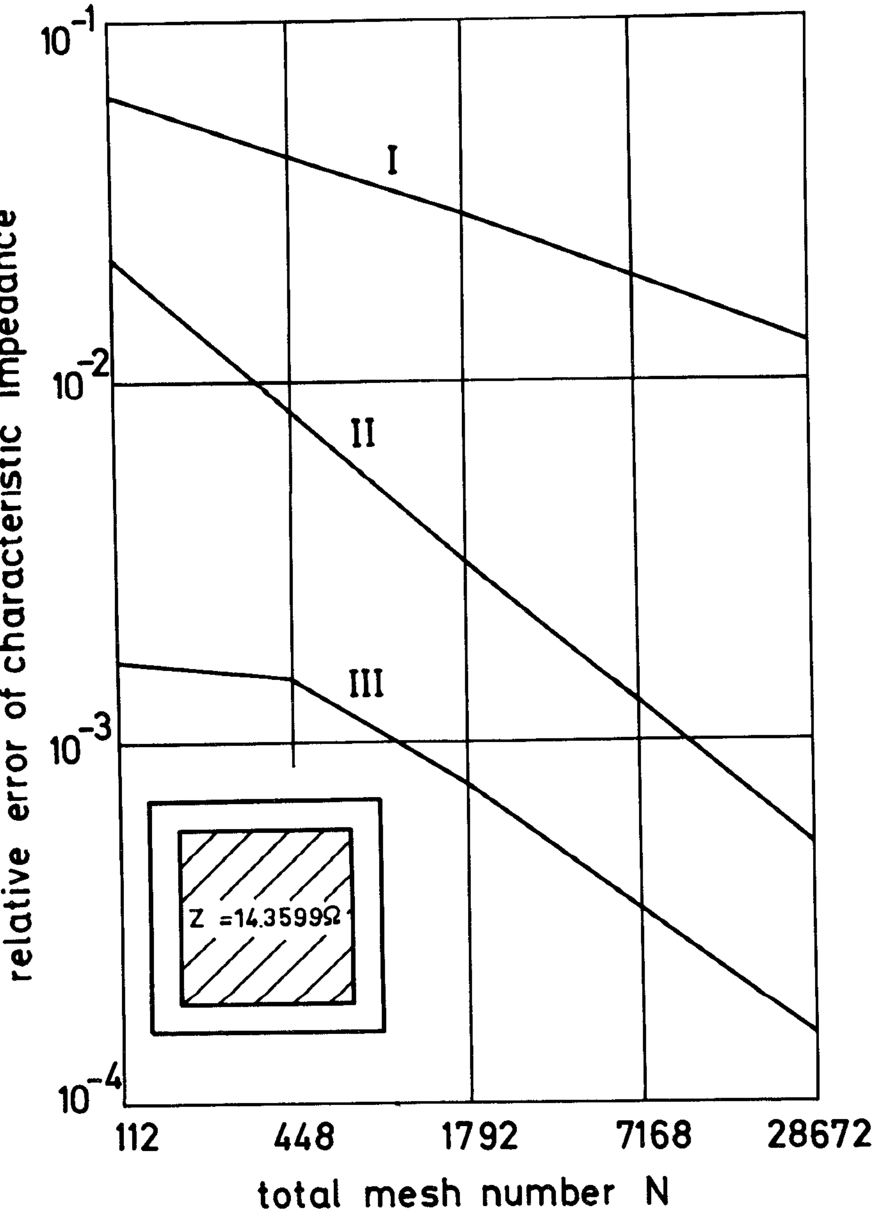
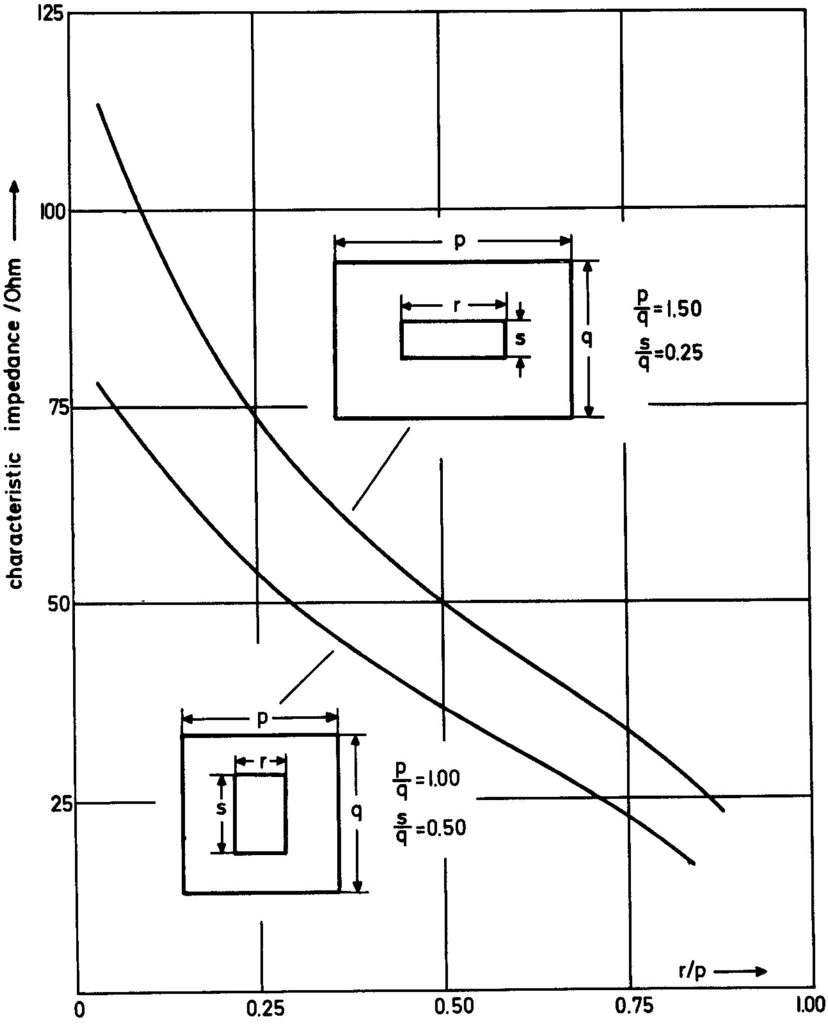


Fig. 1: The error of discretization as a function of the total mesh number

Fig. 2 The characteristic impedance of shielded strip transmission lines



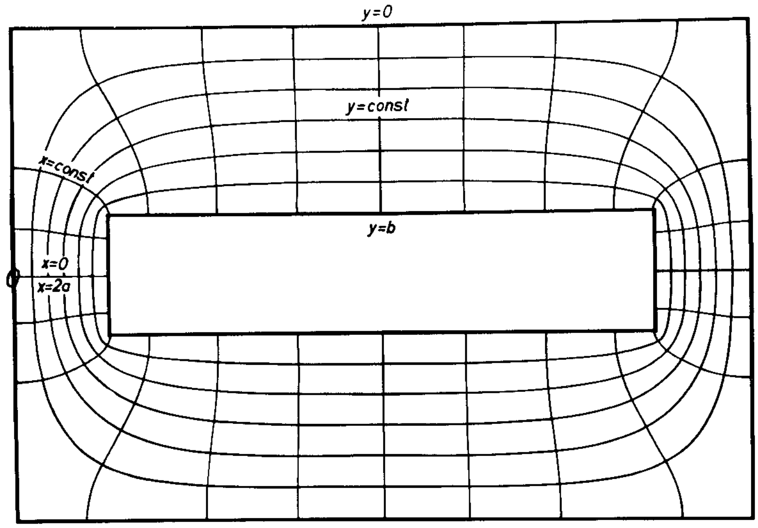


Fig. 3: The curvilinear coordinate system

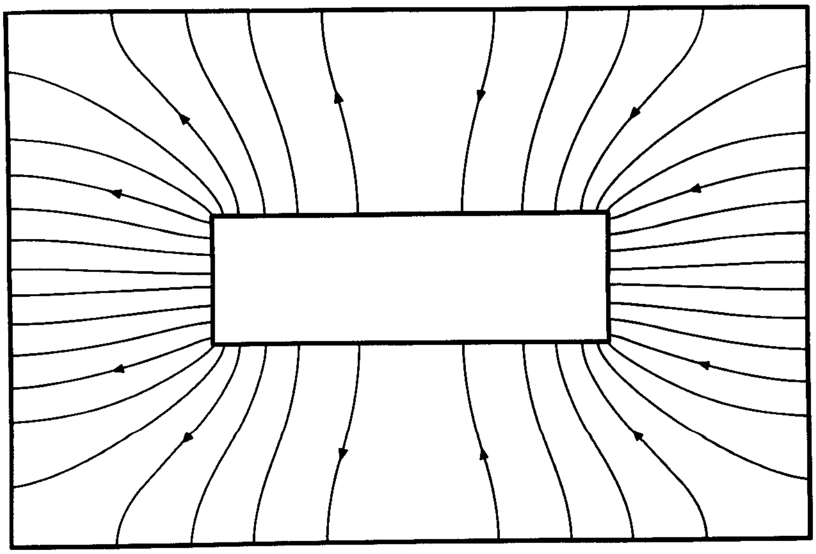


Fig. 4: Transverse electric resp. magnetic field patterns
(a) TE_{01}-mode

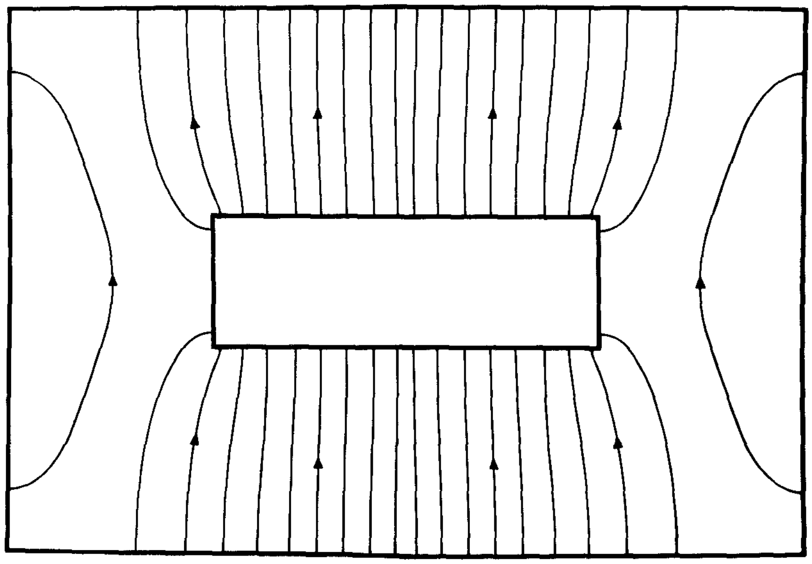


Fig. 4 (b): TE_{1o}-mode

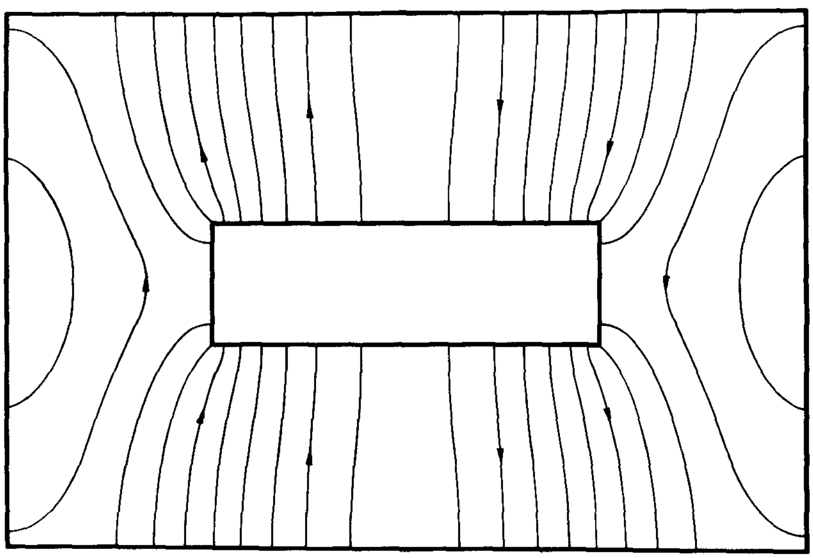


Fig. 4 (c): TE_{2o}-mode

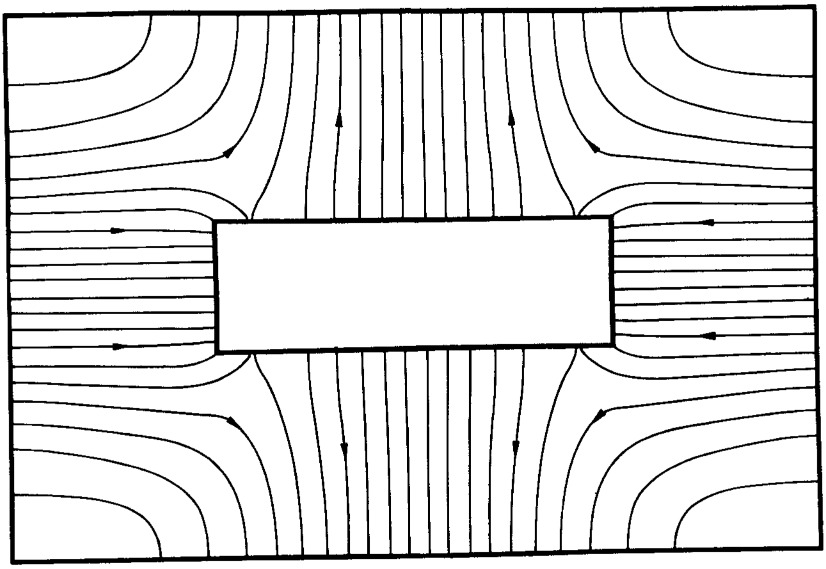


Fig. 4 (d): TE_{11}-mode

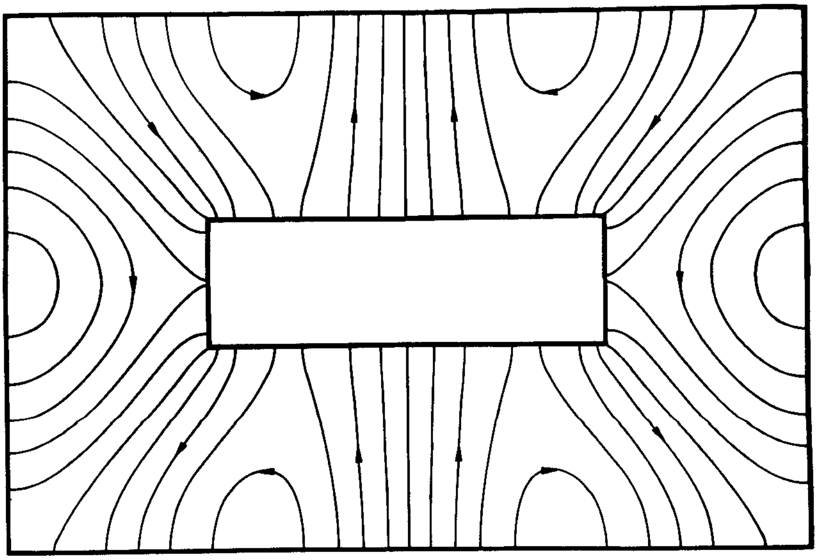


Fig. 4 (e): TE_{3o}-mode

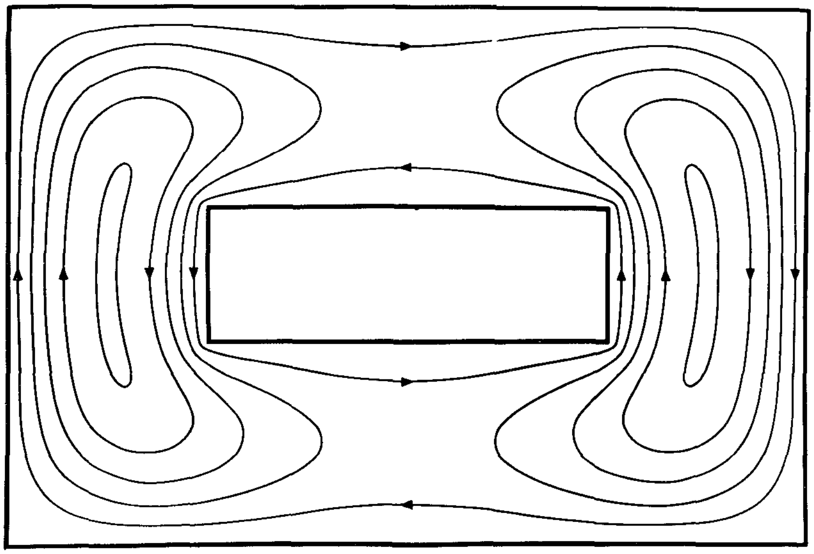


Fig. 4 (f): TM_{11}-mode

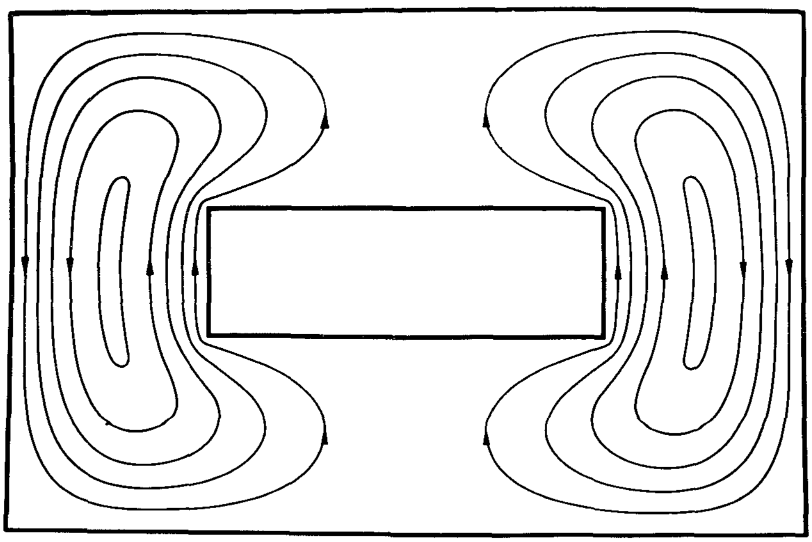


Fig. 4 (g): TM_{21}-mode

Fig. 5a The critical wavelengths of the most important TE-modes

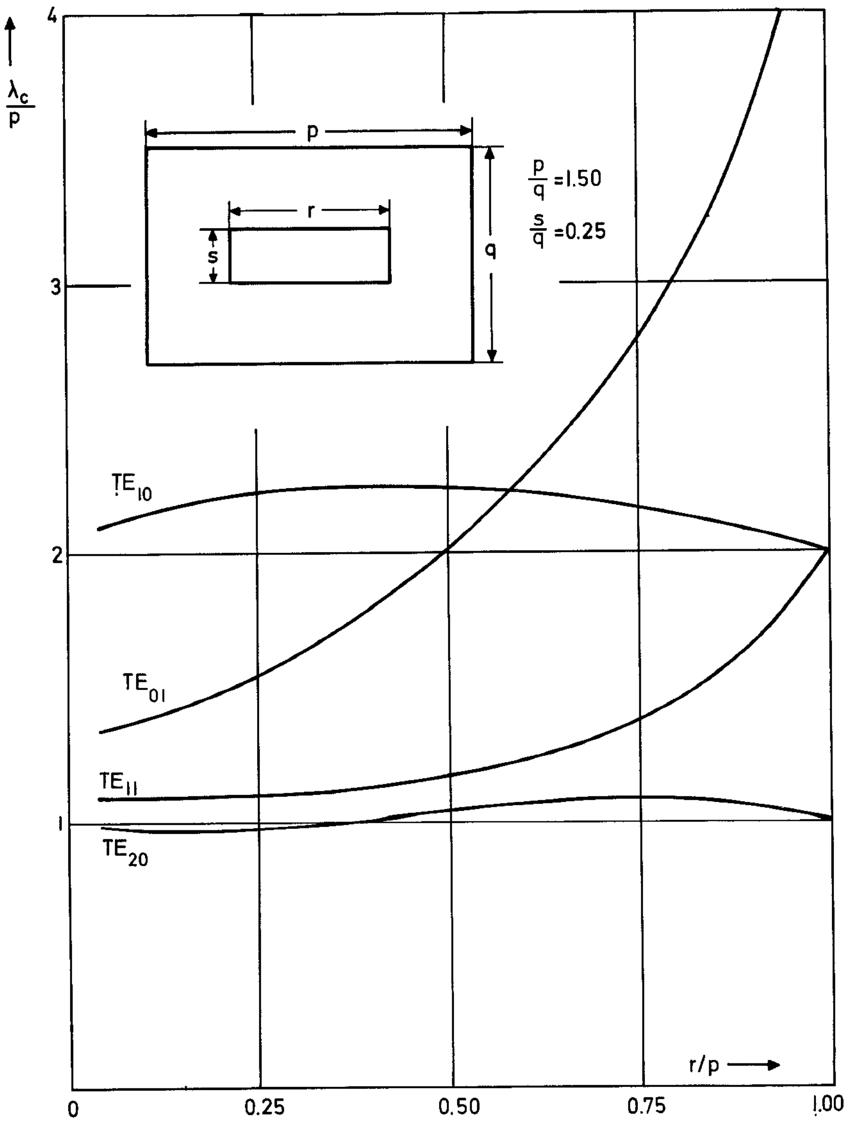


Fig. 5b: The critical wavelengths of the most important TM-modes